1

* ~~$1+e^{i\alpha} = 2\ln\left(\frac{\alpha}{2}\right)e^{i\frac{\alpha}{2}}$~~
* $e^{i\alpha}+e^{i\beta} = 2\cos\left(\frac{\alpha\ \beta}{2}\right)e^{i\frac{\alpha\beta}{2}}$
* $\lim f(x) - ax = b$
* $\left(1+\frac{a}{b(f)}\right)^{g \to \infty} = e^{\frac{a \times g}{b \cdot f}} \to$ Limite
  $\to \infty$
* $\lim\limits_{x\to+\infty} \frac{\ln^m(v)}{x^n} = 0$
* $\lim\limits_{n\to+\infty} \frac{n^{\frac{1}{n}}}{\ln n} = +\infty$
* $\sin x = 0 \Leftrightarrow x = K\pi$
* $\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + K\pi$
* $\frac{a^n}{n^m} \underset{\infty}{\simeq} a^n \simeq +\infty$
* $\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$
* $\lim\limits_{x\to 0} \frac{\tan x}{x} = 1$
* $\lim\limits_{x\to 0} \frac{1-\ln x}{x^2} = \frac{1}{2}$
* $1+e^{ix} = 2\cos\frac{x}{2}e^{i\frac{x}{2}}$
* $e^{ix} - 1 = 2i\sin\frac{x}{2}e^{i\frac{x}{2}}$

* $1-e^{i\theta} = e^{i\frac{\theta}{2}}\left(e^{-i\frac{\theta}{2}} - e^{i\frac{\theta}{2}}\right)$
$= e^{i\frac{\theta}{2}}\left(-2i\sin\frac{\theta}{2}\right)$ ou $= -i\,2\sin\frac{\theta}{2}e^{i\frac{\theta}{2}}$
$= -2i\sin\frac{\theta}{2}e^{i\frac{\theta}{2}}$ $\quad -i = e^{i\frac{-\pi}{2}}$
$= 2\sin\frac{\theta}{2}e^{i\left(\frac{\theta}{2}+\frac{\pi}{2}+\pi\right)}$ $\quad = 2\sin\frac{\theta}{2}e^{i\left(\frac{\theta}{2}-\frac{\pi}{2}\right)}$

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* Règle d'Hopital :

$$\lim_{x \to ?} \frac{f(x)}{g(x)} = \begin{cases} \to \frac{0}{0} \\ \to FI \\ \to \frac{\infty}{\infty} \end{cases} \Rightarrow \lim_{x \to ?} \frac{f'(x)}{g'(x)}$$

FI — calculable

$$\downarrow$$

$$\lim_{x \to ?} \frac{f''(x)}{g''(x)}$$

* Développement limité : (qd $x \to 0$)

$$\ln(1+x) \simeq x - \frac{x^2}{2}$$

$$e^x \simeq 1 + x + \frac{x^2}{2}$$

$$\cos x \simeq 1 - \frac{x^2}{2}$$

$$\sin x \simeq x - \frac{x^3}{6}$$

$$\tan x \simeq x$$

$(1+x)^\alpha \simeq 1 + \alpha x$ avec α cte et pas inconnu et $\in \mathbb{Q}$

* $\lim_{x \to +\infty} \left(1 + \frac{1}{n}\right)^n = \lim_{x \to +\infty} e^{n \ln(1 + \frac{1}{n})}$

$\uparrow$

m̂ chose pour $\left(1 - \frac{1}{n}\right)^{n\alpha}$ ou $\left(1 + \frac{1}{2n}\right)^{-n}$

* $\left(\frac{n+1}{n-1}\right)^n \to \left(\frac{n-1+1+1}{n-1}\right)^n = \left(1 + \frac{2}{n-1}\right)^n$

* $\cos\theta - i\sin\theta \to (-\theta)$ | $-\cos\theta + i\sin\theta \to (\pi - \theta)$

$-\cos\theta - i\sin\theta \to (\pi + \theta)$ ou $(\theta - \pi)$

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	$-\theta$	$\pi-\theta$	$\pi+\theta$
cos	+	−	−
sin	−	+	−
tan	−	−	+

$\cos(-\theta) = \cos\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π
sin	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0

* $\cos 2x = 2\cos^2 x - 1$
  $\cos 2x = 1 - 2\sin^2 x$
  $\sin 2x = 2\cos x \sin x$
  $\cos(ax+b) \longrightarrow \frac{1}{a}\sin(ax+b)$ ⟹ Primitives
  $\sin(ax+b) \longrightarrow -\frac{1}{a}\cos(ax+b)$

* $e^{i\pi/2} = i$ / $e^{i0} = 1$ / $e^{2k\pi i} = 1 \in \mathbb{R}^+$ / $e^{i\pi} = -1$
  $e^{-i\pi/2} = -i$ / $e^{(2k+1)\pi i} = -1 \in \mathbb{R}^-$ / $e^{k\pi i} = \pm 1 \in \mathbb{R}$

* $z\, i\, 1 \ominus z\, i\, \pi$
  $\hookrightarrow z = 2\cos\frac{\theta}{2}\, e^{i\left(\frac{\theta}{2}\right)} = -2\cos\frac{\theta}{2}\, e^{i\left(\frac{\theta}{2}+\pi\right)}$

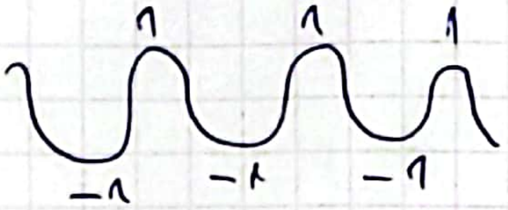
* $e^{i\theta_1} + e^{i\theta_2} = e^{i\left(\frac{\theta_1+\theta_2}{2}\right)}\left(e^{i\left(\frac{\theta_1-\theta_2}{2}\right)} + e^{-i\left(\frac{\theta_1-\theta_2}{2}\right)}\right)$

* $\sum_{k=p}^{n} x^p = x^p + x^{p+1} + x^{p+2} + \dots + x^n$
  $= x^p\left(\frac{1-x^{n-p+1}}{1-x}\right)$

* $\sin(k\pi) = 0$ / $\cos(2k\pi) = 1$ / $\cos((2k+1)\pi) = -1$

* $(-1)^n$: $1 \searrow -1 \nearrow 1 \searrow -1 \nearrow$

cos x
sin x



$\rightarrow$ n'admettent pas de limite (se sont des suites périodiques entre 2 valeurs **fixes**)

* (U_n) Divergente
  - $\rightarrow$ (U_n) n'admet pas de limite.
  - $\rightarrow$ $\lim\limits_{n\to+\infty} U_n = \pm\infty$

* $A = 1 + \cos x + \cos 2x + \dots + \cos nx$

$B = \sin x + \sin 2x + \dots + \sin nx$

$Z = A + iB$

$= 1 + \cos x + i\sin x + \dots + \cos nx + i\sin nx$

$= 1 + e^{ix} + e^{2ix} + \dots + (e^{ix})^n$

$$\boxed{Z = (e^{ix})^0 \frac{1 - (e^{ix})^{n+1}}{1 - e^{ix}}}$$

et $Z = \dfrac{1 - e^{(n+1)xi}}{1 - e^{ix}} = e^{\frac{nxi}{2}} \dfrac{\sin\left(\frac{n+1}{2}x\right)}{\sin\left(\frac{x}{2}\right)}$

* Suites particulières:

* $U_{n+1} = U_n \times q \longrightarrow U_n = U_p \times q^{n-p}$

* $U_{n+1} = q\,\underset{\Uparrow \atop \omega}{U_n} + r \longrightarrow U_n - \omega = (U_p - \omega)\,q^{n-p}$

on pose ω

$\hookrightarrow \omega = q\omega + r \Rightarrow \omega = \dfrac{r}{1-q}$

* $U_{n+1} = U_n^{\,q} \longrightarrow U_n = U_p^{\,q^{n-p}}$

* $U_{n+1} = U_n + r \longrightarrow U_n = U_p + r(n-p)$

$U_0 = \frac{3}{2}$ et $U_{n+1} = \frac{1}{\sqrt{U_n - 1}} - 1$: $\lim U_n = ?$

$$U_{n+1} = \frac{1}{\sqrt{U_n - 1}} + 1 \Rightarrow \underbrace{U_{n+1} - 1}_{V_{n+1}} = \frac{1}{\sqrt{U_n - 1}}$$

$$\Rightarrow V_{n+1} = (\underbrace{U_n - 1}_{V_n})^{-1/2}$$

$$\Rightarrow V_{n+1} = V_n^{-1/2}$$

$$\Rightarrow V_n = V_0^{(-\frac{1}{2})^n} = \left(\frac{1}{2}\right)^{(-\frac{1}{2})^n}$$

$$\Rightarrow U_n = V_n + 1 = \left(\frac{1}{2}\right)^{(-\frac{1}{2})^n} + 1$$

$$\left(\frac{1}{2}\right)^0 + 1 = 2$$

Limites particulières :

* $\lim\limits_{n \to +\infty} \frac{3 \times 5^n - 7 \times 2^n}{5^n + 4 \times 3^n} = \lim \frac{5^n(3 - 7(\frac{2}{5})^n)}{5^n(1 - 4 \times (\frac{3}{5})^n)} = 3$

* $\lim\limits_{n \to +\infty} \frac{(-1)^n n + 3}{5(-1)^n + 2n}$

$$U_{2n} = \frac{(-1)^2\, 2n + 3}{5(-1)^{2n} + 4n} = \frac{2n+3}{5+4n} \to \frac{2n}{4n} \Rightarrow \frac{1}{2}$$

$$U_{2n+1} = \frac{(-1)^{2n+1}(2n+1) + 3}{5(-1)^{2n+1} + 2(2n+1)} = \frac{-2n-2}{-5+4n+2} = \frac{-2n}{4n} = -\frac{1}{2}$$

$$\frac{1}{2} \neq -\frac{1}{2}$$

$\Rightarrow$ n'admet pas de limite.

* $\lim U_{2n} = l_1$ et $\lim U_{2n+1} = l_2$

$l_1 = l_2 = l \Longrightarrow \lim U_n = l$

$l_1 \neq l_2 \longrightarrow (U_n)$ n'admet pas de limite

* $\lim\limits_{x\to ?} \text{Poly}.\, e^{?\to -\infty} = 0$

$\lim\limits_{x\to ?} \dfrac{\text{Poly}}{\text{Poly}}\, e^{?\to -\infty} = 0$

* $I_n = \int_0^1 x^n f(x)\,dx \xrightarrow[n+\infty]{} 0$; $I_n = \int_0^1 \dfrac{x^n}{x+1}\,dx \xrightarrow{n\to+\infty} 0$

* Monotonie de I_n :

$I_{n+1} - I_n = \int_0^1 x^{n+1} f(x) - \int_0^1 x^n f(x)$

$0 \leqslant x \leqslant 1 \qquad = \int_0^1 x^{n+1} f(x) - x^n f(x)$

$= \int_0^1 \underbrace{x^n}_{\geqslant 0} f(x) \underbrace{(x-1)}_{\leqslant 0}$

$\Rightarrow$ le signe de $I_{n+1} - I_n$ dépend de $f(x)$

* Les somme :

* $\sum\limits_{k=p}^{n} U_k = U_p + U_{p+1} + \dots + U_n$

* $\sum\limits_{k=p}^{n} \lambda U_k = \lambda \sum\limits_{k=p}^{n} U_k$

* $\sum\limits_{k=p}^{n} (U_k + V_k) = \sum U_k + \sum V_k$

* $\sum\limits_{k=p}^{n} U_n \times V_n \neq \sum U_n \times \sum V_n$

* $\sum\limits_{k=p}^{n} \lambda = \lambda \times (n - p + 1)$

* $\sum_{K=p}^{n} K = p + (p+1) + (p+2) + \cdots + n = \frac{(p+n)}{2} \times (n-p+1)$

* $\sum_{K=p}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}$; $\sum_{K=1}^{10} K^2 = \frac{10 \times 11 \times 21}{6}$

$\sum_{K=p}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2$ $= 55 \times 7 = 385$

* $\sum_{K=p}^{n} U_{(K+1)} - U_k = U_{n+1} - V_n$

$\sum_{K=2}^{100} \ln(K-1) - \ln K = \ln(2-1) - \ln 100 = -\ln 100$

$\sum_{K=1}^{100} \frac{1}{K} - \frac{1}{K+1} = 1 - \frac{1}{101}$

(lKbir avec n et sghir avec K)

* $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

* $\int e^{\alpha x} \cos \beta x$ ou $\int e^{\alpha x} \sin \beta x$

$\Rightarrow \int e^{\alpha x} e^{\beta x i} = \int e^{(\alpha + \beta i)}$

$= \left[e^{(\alpha + \beta i) x} \right] \times \frac{1}{\alpha + \beta i}$

$= \frac{1}{\alpha^2 + \beta^2} (\alpha - \beta i) [? + ?i]$

$= \underset{\cos}{\underset{\downarrow}{A}} + \underset{\sin}{\underset{\downarrow}{B}} i$

Exple : $\int_0^{\frac{\pi}{2}} \sin 2x \; e^x$

$\Rightarrow \int_0^{\frac{\pi}{2}} e^x e^{2xi} = \int_0^{\frac{\pi}{2}} e^{2xi+x} = \int_0^{\frac{\pi}{2}} e^{x(1+2i)}$

$= \frac{1}{1+2i}\left[e^{(1+2i)x}\right]$

$= \frac{1}{5}(1-2i)\left(e^{(1+2i)\frac{\pi}{2}} - 1\right)$

$e^{\pi/2} \times e^{i\pi}$, $e^{i\pi} = -1$

$= \frac{1}{5}(1-2i)\left(-e^{\frac{\pi}{2}} - 1\right)$

$\hookrightarrow \frac{1}{5}(-2i)\left(-e^{\pi/2} - 1\right)$

$\frac{2}{5}\left(e^{\pi/2} + 1\right)$

* $f(x) = \cancel{x}\, e^{-x}$

$\rightarrow F(\cancel{x}) = (ax+b)e^{-x}$

$\rightarrow F'(x) = (-ax + a - b) = xe^{-x}\, f(x)$

$F(x) = (-x-1)e^{-x}$

$I = F(\ln(2)) - F(0)$

$= (-\ln 2 - 1)\frac{1}{2} + 1$

$\begin{cases} -a = 1 \Rightarrow a = -1 \\ a - b = 0 \\ a = b = -1 \end{cases}$

* $I = \int_2^3 1 \ln(x^2 - 1)\, dx$

$\Rightarrow \begin{cases} u = \ln x^2 - 1 \\ v' = 1 \end{cases} \Longrightarrow \begin{cases} u' = \frac{2x}{x^2-1} = \frac{2x}{(x-1)(x+1)} \\ V = x \end{cases}$

$I = [\quad] - \int_2^3 \frac{2x^2}{x^2-1}$ $\qquad 2x^2 \,\big|\, \frac{x^2-1}{2}$

$= [\quad) - \int_2^3 \frac{2(x-1)+2}{x-1} = \int_2^3 2 + \frac{2}{x-1}$

(on ajoute et on retire)

* $I = \int_2^3 \frac{x+1}{x^2+x-2} = \int_2^3 \frac{x+1}{(x+2)(x-1)}$

$\int_2^3 \frac{a}{(x-1)} + \frac{b}{x+2}$

On remplace $\Downarrow$ $x = 1$

$\leftarrow$?

* égalité = t3wid.

* $z \in \mathbb{R} \Leftrightarrow Im(z) = 0 \Leftrightarrow z = \bar{z}$

* $z \in i\mathbb{R} \Leftrightarrow R_e(z) = 0 \Leftrightarrow z = -\bar{z}$

* $|z - z_A| = |z - z_B| \rightarrow$ la médiatrice de (AB)

* $|z - z_\Omega| = R \rightarrow$ le cercle de centre Ω et de rayon R

* $\begin{cases} ax + by + c = 0 \\ \text{et} \\ x \geq \alpha \end{cases} \Rightarrow$ Demi droite ou $y \lessgtr b$

* $\begin{cases} (x-a)^2 + (y-b)^2 = R \\ \text{et} \\ x \gtrless a \text{ ou } y \gtrless b \end{cases} \Rightarrow$ Demi cercle

- $\begin{cases} (x-a)^2 + (y-b)^2 = R^2 \\ x > a \quad \text{et} \quad y < b \end{cases} \Rightarrow \frac{1}{4}$ de cercle

- $(x-a)^2 + (y-b)^2 < R^2 \Rightarrow$ disque